

# Contents

<b>Foreword to the Second Edition .....</b>	<b>11</b>
<b>Prologue .....</b>	<b>13</b>
<b>Introduction .....</b>	<b>17</b>
1 The Sudoku problem and the resolution methods.....	17
2 The roles of Logic and AI in this book.....	25
3 Examples and classification results .....	27
<b>PART ONE: FOUNDATIONS .....</b>	<b>33</b>
<b>Chapter I. Symmetries, analogies and supersymmetries.....</b>	<b>29</b>
I.1 Symmetries.....	35
I.2 The two canonical coordinate systems on a grid.....	37
I.3 Coordinates and names.....	38
I.4 Analogies.....	39
I.5 Supersymmetries.....	40
<b>Chapter II. Complementary graphical representations .....</b>	<b>41</b>
II.1 New graphical representations of a puzzle .....	41
II.2 Extended Sudoku Board .....	45
II.3 How to build the rn- and cn- representations .....	47
II.4 How to use the rn- and cn- representations .....	49

<b>Chapter III. Grid Theory and Sudoku Theory.....</b>	<b>51</b>
III.1 Multi-sorted first order logic and Sudoku Grid Theory .....	52
III.2 Sudoku Theory (ST).....	60
<b>Chapter IV. Sudoku Resolution Theories.....</b>	<b>65</b>
IV.1 Sudoku Theory vs. Sudoku Resolution Theories .....	66
IV.2 The logical nature of Sudoku Resolution Theories.....	67
IV.3 Basic Sudoku Resolution Theory (BSRT) .....	75
IV.4 Block-free predicates and formulæ.....	81
IV.5 Symmetries and the three basic meta-theorems .....	87
<b>PART TWO: BASIC RESOLUTION RULES.....</b>	<b>91</b>
<b>Chapter V. Subset rules, level one: Singles .....</b>	<b>93</b>
V.1 Subset rules, level one.....	93
V.2 Logical formulation of the rules .....	95
V.3 Example .....	95
V.4 Theory L1_0 .....	97
<b>Chapter VI. Subset rules, level two: Pairs.....</b>	<b>99</b>
VI.1 Naked-Pairs .....	101
VI.2 Hidden-Pairs .....	104
VI.3 Super-Hidden-Pairs or X-Wing.....	110
VI.4 Theory L2 .....	129
<b>Chapter VII. Subset rules, level three: Triplets .....</b>	<b>121</b>
VII.1 Naked-Triplets .....	122
VII.2 Hidden-Triplets .....	128
VII.3 Super-Hidden-Triplets or Swordfish .....	130
VII.4 Theory L3_0.....	134
<b>Chapter VIII. Subset rules, level four: Quadruplets .....</b>	<b>135</b>
VIII.1 Naked-Quadruplets .....	137
VIII.2 Hidden-Quadruplets .....	140
VIII.3 Super-Hidden-Quadruplets or Jellyfish .....	141
VIII.4 Correspondence between Naked and Hidden subsets.....	147
VIII.5 Theory L4_0 .....	147

<b>Chapter IX. Interaction Rules .....</b>	<b>149</b>
IX.1 Row-Interaction-with-Block .....	149
IX.2 Column-Interaction-with-Block .....	151
IX.3 Block-Interaction-with-Row and Block-Interaction-with-Column .....	153
IX.4 How the interaction rules integrate our hierarchy .....	155
IX.5 Examples and independence results .....	155
IX.6 Theory L1 .....	159
<b>Chapter X. The XY-Wing and XYZ-Wing rules .....</b>	<b>161</b>
X.1 XY-Wing or XYW .....	161
X.2 XYZ-Wing or XYZW .....	166
X.3 Theory L3 .....	169
X.4 Theories L1_0 to L4_0 .....	170
<b>PART THREE: 2D CHAIN RULES.....</b>	<b>175</b>
<b>Chapter XI. General theorems on shared units .....</b>	<b>177</b>
XI.1 Shared units in natural row-column space .....	177
XI.2 Hidden counterparts of the previous theorems .....	180
<b>Chapter XII. Chains, target cells and chain rules.....</b>	<b>183</b>
XII.1 Links, chains, targets and chain rules .....	184
XII.2 xy-links, xy-chains and xy-chain rules .....	188
XII.3 c-links, c-chains and c-chain rules .....	191
XII.4 Formal interpretation of the rules and of their proofs .....	195
<b>Chapter XIII. Graphico-logical patterns for chain rules .....</b>	<b>197</b>
XIII.1 Simple patterns for cells and chains; their graphical representations and their instantiations .....	197
XIII.2 Logical formula associated to a simple chain pattern.....	201
XIII.3 Graphico-logical expression of chain rules .....	204
<b>Chapter XIV. xy-chains .....</b>	<b>207</b>
XIV.1 Why one should not allow loops in xy-chains .....	207
XIV.2 List of the first xy-chains .....	211
XIV.3 Examples and independence results .....	214

<b>Chapter XV. Hidden xy-chains (hxy-chains)</b> .....	<b>225</b>
XV.1 Introduction to hidden xy-chains (or hxy-chains) .....	225
XV.2 The unifying power of hidden xy-chains .....	230
XV.3 Examples and independence results .....	233
<b>Chapter XVI. Conjugacy chains (c-chains)</b> .....	<b>245</b>
XVI.1 Why one should not allow loops in c-chains.....	246
XVI.2 Special c-chains and the case of hidden c-chains .....	248
XVI.3 Examples of c4-chains; independence of C4, XY4 and HXY4 .....	250
XVI.4 Example of a c6-chain .....	254
XVI.5 Example of a c8-chain .....	256
<b>Chapter XVII. xyt-chains</b> .....	<b>257</b>
XVII.1 xyt-chains and xyt-chain rules .....	257
XVII.2 Extended cell patterns and chain patterns .....	260
XVII.3 List of the first xyt-chain rules.....	262
XVII.4 Examples and independence results .....	264
<b>Chapter XVIII. Hidden xyt-chains (hxyt-chains)</b> .....	<b>279</b>
XVIII.1 Introduction to hidden xyt-chains (or hxyt-chains).....	279
XVIII.2 Examples and independence results .....	281
<b>Chapter XIX. xyz- and xyzt- chains</b> .....	<b>297</b>
XIX.1 Introduction to xyz-chains .....	297
XIX.2 Introduction to xyzt-chains.....	302
XIX.3 List of the first xyzt-chain rules .....	305
XIX.4 Examples and independence results.....	307
<b>Chapter XX. Hidden xyz- and xyzt- chains (hxyz- and hxyzt- chains)</b> .....	<b>315</b>
XX.1 Introduction to hidden xyzt-chains (or hxyzt-chains) .....	315
XX.2 Examples and independence results .....	318
<b>Chapter XXI. Classification results</b> .....	<b>323</b>
XXI.1 Detailed results for levels L0 to L4_0 .....	324
XXI.2 Global results for levels L4_0 to L13 .....	326
XXI.3 Further results for levels L4_0 to L7 .....	327
XXI.4 Comparison with number of entries (Sudogen0) .....	329

**PART FOUR: FULLY SUPERSYMMETRIC (OR "3D") CHAIN RULES 331**

<b>Chapter XXII. 3D chains: nrc-, nrct-, nrcz- and nrczt- chains .....</b>	<b>333</b>
XXII.1 nrc-cells, candidates, nrc-links and nrc-chains .....	334
XXII.2 nrc-chains .....	337
XXII.3 nrct-chains .....	338
XXII.4 nrcz-chains .....	340
XXII.5 nrczt-chains .....	341
XXII.6 Proof of the nrc-, nrct-, nrcz- and nrczt- chain rules .....	342
XXII.7 Graphico-logical patterns for 3D chains and associated logical formulae .....	343
XXII.8 Miscellanea .....	343
<b>Chapter XXIII. Examples and classification results for 3D chains .....</b>	<b>345</b>
XXIII.1 3D chains: priorities and notation .....	345
XXIII.2 Examples of 3D chains of types nrc, nrct, nrcz and nrczt .....	348
XXIII.4 Classification results .....	363
XXIII.5 An unsolved puzzle: Easter Monster .....	364
<b>Chapter XXIV. Resolution rules and resolution techniques .....</b>	<b>365</b>
XXIV.1 On the concept of a resolution rule .....	365
XXIV.2 Chains and their objective properties .....	369
XXIV.3 Resolution rule versus resolution techniques .....	373
XXIV.4 Several resolution techniques implementing the nrczt-chain rule .....	375
XXIV.5 The Trial and Error Theorem .....	379
<b>Miscellanea .....</b>	<b>385</b>
1 The question of completeness .....	385
2 The question of confluence .....	387
3 The question of uniqueness .....	391
4 Are any other types of rules necessary? .....	401
<b>Conclusion .....</b>	<b>403</b>
<b>References.....</b>	<b>413</b>